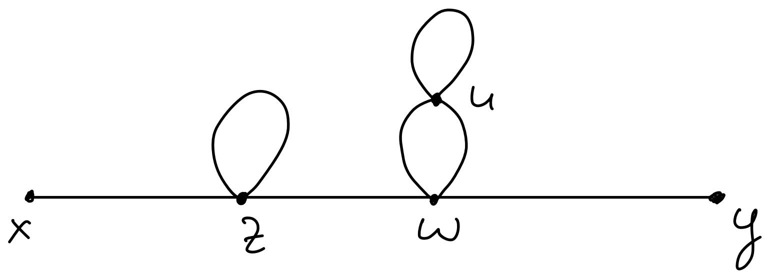
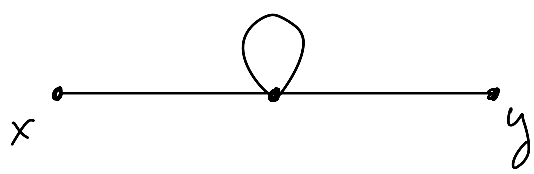
Last time we wrote down the expression for diagram

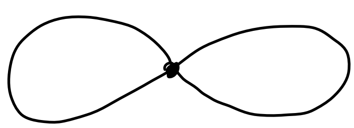


$\rightarrow$ found that this has symmetry factor $S = 8 = 2 \cdot 2 \cdot 2$

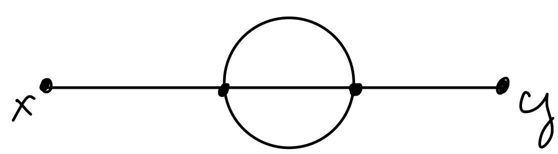
Let's give a couple more examples:



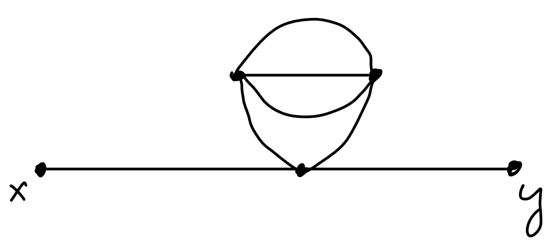
$S = 2$



$S = 2 \cdot 2 \cdot 2 = 8$



$S = 3! = 6$



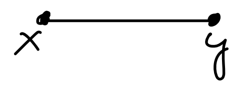
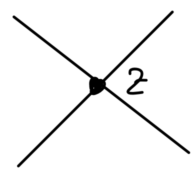
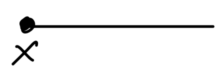
$S = 3! \cdot 2 = 12$

The Feynman rules

Draw diagrams for all possible Wick contractions in

$$G^{(N)}(x_1, x_2, \ldots, x_N) = \frac{1}{Z[0]} \frac{\delta^N Z[J]}{\delta J(x_1) \cdots \delta J(x_N)} \Bigg|_{J=0}$$

In φ^4-th, assign

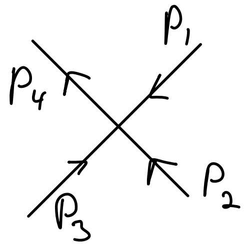
1) for each propagator,  $= D_F(x-y)$
2) for each vertex,  $= (-i\lambda)\int d^4 z$
3) for each external point  $= 1$
4) divide by the symmetry factor

$\longrightarrow$ "position-space" Feynman rules

In most calculations, it is simpler to express Feynman rules in terms of momenta:

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{-i}{p^2 - m^2 + i\varepsilon} e^{ip\cdot(x-y)}$$

→ when 4 lines meet at a vertex,

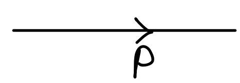


$\longleftrightarrow$

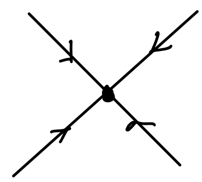
$$\int d^4 z \; e^{ip_1 \cdot z} e^{ip_2 \cdot z} e^{ip_3 \cdot z} e^{-ip_4 \cdot z}$$
$$= (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p_4)$$

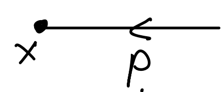
momentum is "conserved" at each vertex

→ "momentum-space" Feynman rules:

1) for each propagator, 

$= \dfrac{-i}{p^2 - m^2 + i\varepsilon}$

2) for each vertex,  $= -i\lambda$

3) for each external point, 

$= e^{ip\cdot x}$

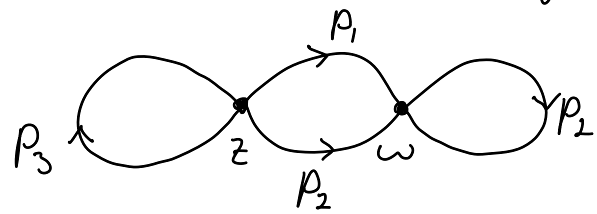
4) impose momentum conservation at each vertex

5) integrate over each undetermined momentum: $\displaystyle\int \frac{d^4 p}{(2\pi)^4}$

6) divide by the symmetry factor

the integral $\int d^4 p$ is a result of the superposition principle of quantum mechanics.

Consider now the Feynman diagram:

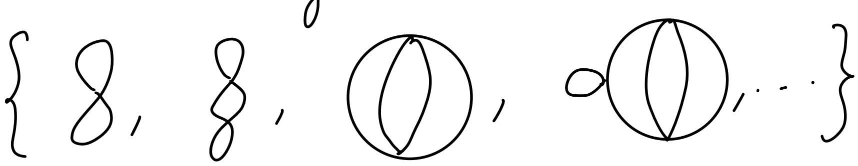


→ delta function for left-hand vertex:
$$(2\pi)^4\, \delta^{(4)}(p_1+p_2)$$

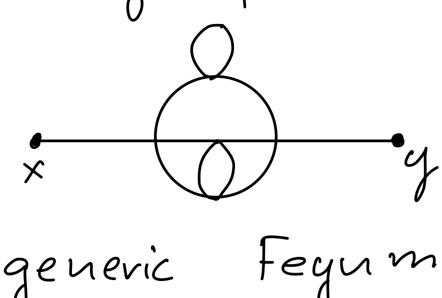
→ integrating results in delta function for right-hand vertex: $\underbrace{(2\pi)^4\, \delta^{(4)}(0)}_{\text{interpret as volume of space-time}}$

→ "disconnected" diagram

Disconnected diagrams can have the form

$V_i \in$ 

whereas a "connected" diagram is typically of the form



A generic Feynman diagram consists of one connected and several disconnected pieces suppose it has n_i pieces of the form V_i

$\rightarrow$ value of diagram:

$$(\text{value of connected piece}) \cdot \prod_i \frac{1}{n_i!}(V_i)^{n_i}$$

symmetry factor arising from interchange of pieces

$$\rightarrow \sum_{\substack{\text{all possible}\\ \text{connected pieces}}} \sum_{\text{all } \{n_i\}} \begin{pmatrix}\text{value of}\\ \text{connected piece}\end{pmatrix} \times \left(\prod_i \frac{1}{n_i!}(V_i)^{n_i}\right)$$

$$= \left(\sum \text{connected}\right) \times \sum_{\text{all } \{n_i\}} \left(\prod_i \frac{1}{n_i!}(V_i)^{n_i}\right)$$

$$= \left(\sum \text{connected}\right) \times \left(\sum_{n_1} \frac{1}{n_1!} V_1^{n_1}\right)\left(\sum_{n_2} \frac{1}{n_2!} V_2^{n_2}\right)\left(\sum_{n_3} \frac{1}{n_3!} V_3^{n_3}\right)\cdots$$

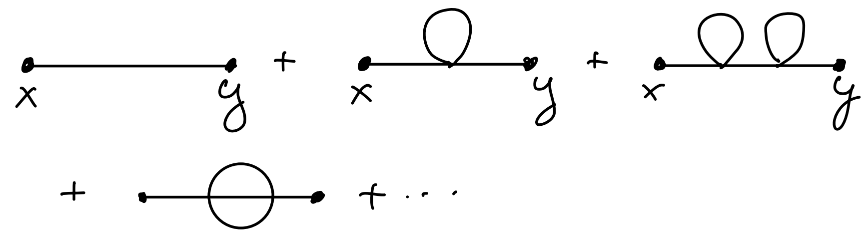
$$= \left(\sum \text{connected}\right) \times \prod_i \exp(V_i)$$

$$= \left(\sum \text{connected}\right) \times \underbrace{\exp\left(\sum_i V_i\right)}_{= Z[0]}$$

Let us now consider

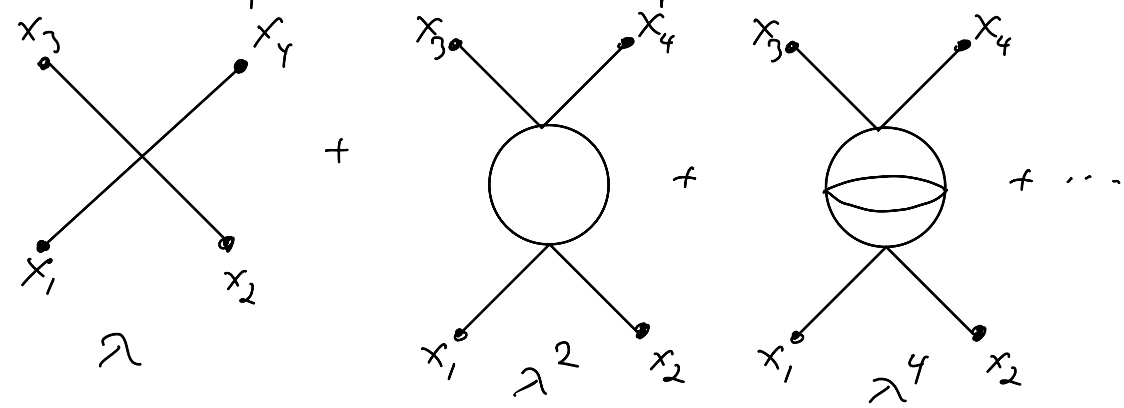
$G^{(2)}(x, y)$

= sum of all connected diagrams with two external points

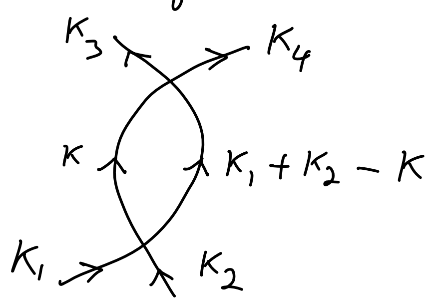
= 

$G^{(4)}(x_1, x_2, x_3, x_4)$

= sum of all connected diagrams with four external points

= 

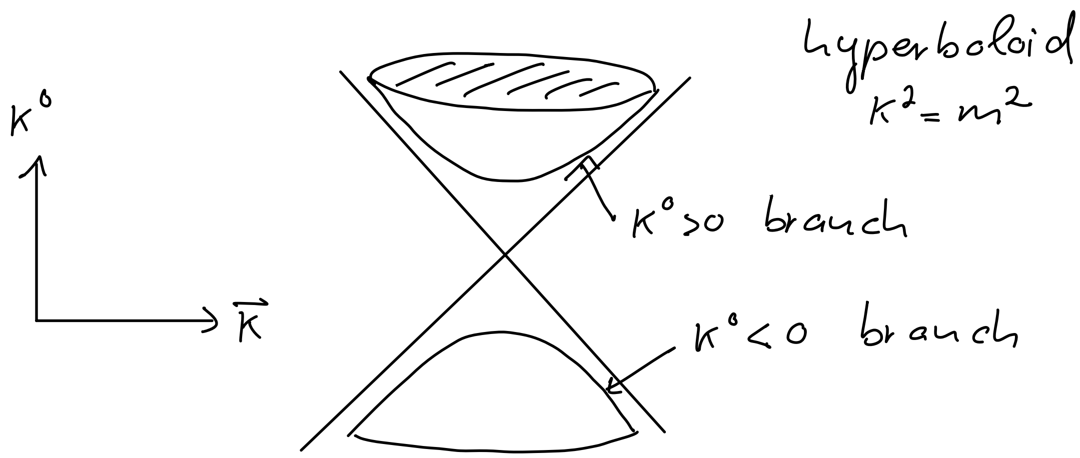
$\longrightarrow$ one-loop diagram in momentum space:



→ applying the Feynman rules, we obtain

$$\frac{1}{2}(-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{(k_1+k_2-k)^2-m^2+i\varepsilon}$$

the two particles with momenta k and k_1+k_2-k running in the loop are "virtual particles" as their momentum is <u>not</u> restricted to be on "mass-shell."

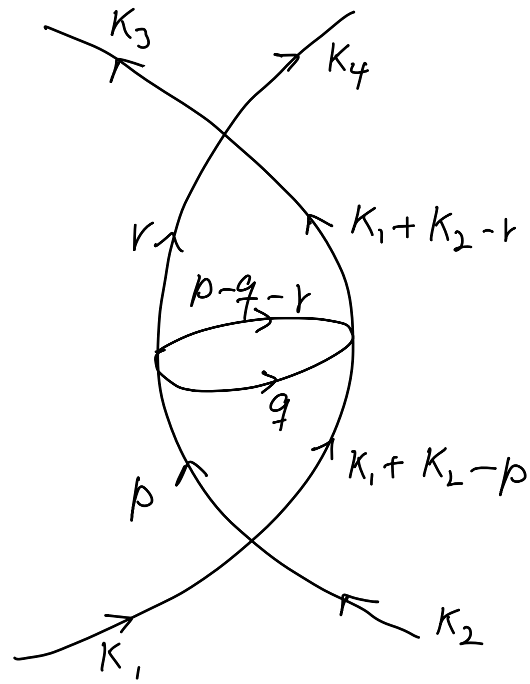


For large k the integrand goes as $\frac{1}{k^4}$.

→ integral is divergent

Will see how to fix such divergences later!

Looking at the 3-loop diagram



$\longrightarrow$ of order $O(\lambda^4)$

not worrying about symmetry factors, we get

$$(-i\lambda)^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} \frac{i}{(k_1+k_2-p)^2-m^2+i\varepsilon}$$

$$\times \frac{i}{q^2-m^2+i\varepsilon} \frac{i}{(p-q-r)^2-m^2+i\varepsilon}$$

$$\times \frac{i}{r^2-m^2+i\varepsilon} \frac{i}{(k_1+k_2-r)^2-m^2+i\varepsilon}$$

again, this triple integral also diverges: it goes as $\int d^{12} \frac{p}{p^{12}}$